Analysis of Vestibulo-ocular Reflex by Evolutionary Algorithm

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ABSTRACT: In this paper the problem of analysis of eye movements using sinusoidal head rotation test is presented. The reflex generated by the rotational sinusoidal test is known as vestibulo-ocular reflex (VOR) producing nystagmus, which consists of slow and fast component. The goal of the method is to discard automatically the effect of the fast phase-saccades and consequently calculate the response of vestibular system in the form of phase shift and amplitude. After filtering the saccades out we are left with discontinuous signal segments. This paper presents an approach to align them to form a smooth signal with the same frequencies that were originally present in the source signal. The approach is based on a direct estimation of the signal component parameters. Two methods of direct search are compared: the Nelder-Mead simplex search and the evolutionary strategy with covariance matrix adaptation. The experimental evaluation on artificial and real-world signals revealed that the evolutionary strategy is more robust, scalable and reliable method, however, its success strongly depends on the saccades removal algorithm.

KEYWORDS: vestibulo-ocular reflex, head rotation test, signal processing, evolutionary algorithm, evolutionary strategy, covariance matrix adaptation, Nelder-Mead simplex search

INTRODUCTION

Vestibulo-ocular reflex (VOR) is responsible for maintaining retinal image stabilization in the eyes during relatively brief periods of head movement [1]. By analyzing the VOR signal, physicians can recognize some pathologies of the vestibular organ which may result in e.g. failures of the balance of a patient. A decision support system that would allow the physicians to diagnose the pathologies of the vestibular organ more precisely and in time needs a method for estimation of the frequency characteristic of the vestibular organ.

Servocontrolled rotating systems have been used as the only practical method to generate stimuli and to test functionality of vestibular organ. The principle of the frequency response measurement using servocontrolled mechanism is relatively simple: the patient is situated in a chair which is then rotated in a defined way following a source signal—sine wave or a mixture of sine (MOS) waves. The chair with the patient is situated in the dark, the patient performs some mental tasks which should distract him from mental visualization that could prevent the eye movements which are subsequently measured. This is called the head rotation test. Since the resulting eye signal is distorted by fast eye movements, so-called saccades, they must be removed from the signal. The vestibulo-ocular reflex is known to respond to rotational frequencies extending below 0.1 Hz [1].

This VOR signal serves as a source for measuring the slow-phase velocity and the frequency response. The measurement of frequency response is usually done on the basis of interpolating these segments with some smooth curves and performing a Fourier transform of the resulting continuous signal. The frequency response created this way contains, however, some artifacts that come from the artificial interpolation curves and are not generated by the vestibular system.

The result of the stimuli is a prevailing pattern called nystagmus consisting of slow and fast phase (see Fig. 1). The first task of this work is the separation of slow and fast phases. Most previous algorithms used to yield slow phase eye velocity were based on least-square method [2] fitted to sinusoid prototype or on syntactic method [3].

The second task of this paper is a method for the direct estimation of the gain and phase lag of the individual sine components of the underlying MOS signal, i.e. for the measurement of several points of the frequency response at the same time. After the estimation, the VOR signal segments should match with the corresponding parts of the estimated MOS signal.

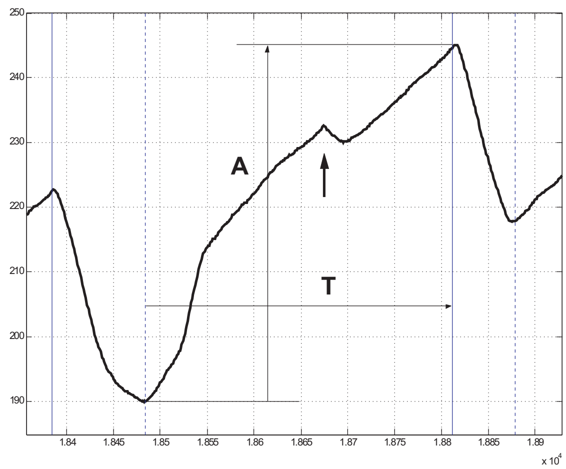

Figure 1: Slow Phase Velocity Definition, $SPV = A/T$

METHODOLOGY

DATA ACQUISITION

Regarding the measurement set up, eye movements were recorded using silver/silver chloride electrodes paled near and lateral each with a reference strip-chart recorder extended by digitized unit. For the eye movements capture the sampling frequency was set at 500 Hz that is considered to be enough for accurate capture of all eye movement characteristics. The rotating frequency of chair was constant throughout recording accelerating or decelerating and changing directions hence producing sinusoidal frequency signal with period 0.05 Hz.

SACCADES FILTERING

Before the slow components are detected, the signal is preprocessed. First, the velocity is computed using derivation of horizontal eye movement channel. Afterwards the signal is filtered out implementing elliptic low pass filter. Taking advantage of signal statistical behaviour, the signal velocity is normalized in such a way that its mean is zero and standard deviation is one. In the next step the detection of slow phase is applied. It is based on horizontal velocity signal; the information of vertical signal is not regarded as useful. The eye movement signal consists of series of triangles that are superposed on the isoline that is most similar to sinusoidal function. Artefacts presented in the signal such as noise are reduced by low pass filtering. Therefore after these assumptions the peaks of horizontal velocity signal detect the position of slow phases. Peaks are simply determinated by setting up the threshold as it is shown in Fig. 2.

PROBLEM SPECIFICATION

It is assumed that the source signal (which controls the rotation of the chair with the patient) is formed as a mixture of sine waves:

$$y^S(t) = \sum_{i=1}^{n} a_i^S \sin(2\pi f_i t + \phi_i^S)\,, \tag{1}$$

where $y(t)$ is the source signal and a_i, f_i and ϕ_i are the amplitude, the frequency and the phase shift of the individual sine components, respectively. The superscript S indicates the relation to the source signal. Note that the frequencies f_i are not marked with this superscript.

Furthermore, it is assumed that the output signal of the vestibular organ is of the same form as the input one, i.e. it contains only sine components with the *same frequencies* as the source signal but possibly with different amplitudes and phase shifts. It should be of the form

$$y(t) = \sum_{i=1}^{n} a_i \sin(2\pi f_i t + \phi_i)\,. \tag{2}$$

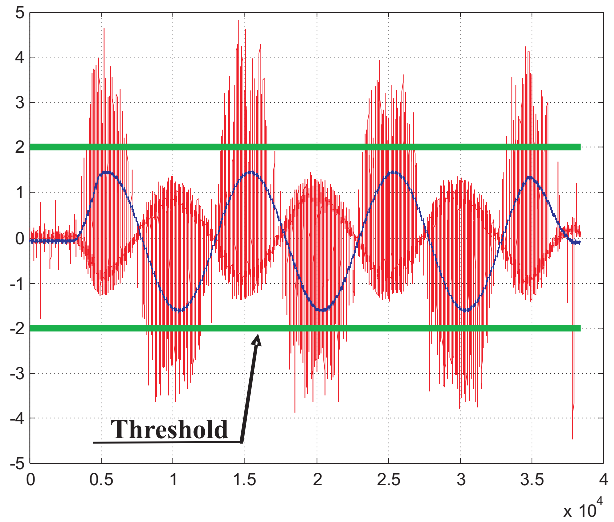

Figure 2: Detection of Fast Phases.

If we knew the a_i and ϕ_i parameters of the output MOS signal components, we could calculate the amplification (a_i/a_i^S) and phase lag $(\phi_i - \phi_i^S)$ at individual frequencies and deduce the state of the vestibular organ.

Unfortunately, we do not have access to the output MOS signal described by Eq. 2. We have only the measured VOR signal, i.e. the segments of the output MOS signal that are left after filtering out the saccades from the eye-tracking signal (see Fig. 6). However, we can search for the unknown parameters a_i and ϕ_i of the output MOS signal by solving the optimization task described in the following text.

MINIMIZING THE LOSS FUNCTION

Let m be the number of segments of the VOR signal at hand, $v_j(t)$, $j = 1 \ldots m$, be the actual j-th segment of the VOR signal and t_j^{ini} and t_j^{end} be the initial and the final time instants for the j-th signal segment. As stated above, we can find the parameters of the output MOS signal by searching the $2n$-dimensional space of points $\mathbf{x}$, $\mathbf{x} = (a_1, \phi_1, \ldots, a_n, \phi_n)$. Such a vector of parameters represents an estimate of the output MOS signal and we can compute the degree of fidelity with which the MOS corresponds to the VOR signal segments by constructing a loss function as follows:

$$L(\mathbf{x}) = \sum_{j=1}^{m} \sum_{i=t_j^{ini}}^{t_j^{end}} \left((v_j(i) - \bar{v}_j) - (y(i) - \bar{y}_j) \right)^2 , \tag{3}$$

where $\bar{v}_j$ is the mean value of the j-th VOR signal segment and is computed as

$$\bar{v}_j = \frac{1}{t_j^{end} - t_j^{ini}} \sum_{i=t_j^{ini}}^{t_j^{end}} v_j(i) , \tag{4}$$

and $\bar{y}_j$ is the mean value of the current estimate of the output MOS signal related to the j-th segment and is computed as

$$\bar{y}_j = \frac{1}{t_j^{end} - t_j^{ini}} \sum_{i=t_j^{ini}}^{t_j^{end}} y(i) . \tag{5}$$

Subtracting the means $\bar{v}_j$ and $\bar{y}_j$ from the VOR signal segments $v_j(t)$ and MOS signal $y(t)$, respectively, we try to match the VOR signal segment to the corresponding part of the MOS signal. If they match, their difference is zero, otherwise it is a positive number quadratically increasing with the difference. This operation is carried out for all m VOR signal segments.

OPTIMIZATION METHODS

The parameter vector $\mathbf{x}$ is projected to the loss function via the estimate of the MOS signal $y(i)$ (and via the mean values $y_j(i)$). In principle, the loss function $L(\mathbf{x})$ is differentiable with respect to the individual parameters. Thus, to find the optimal values of the parameter vector $\mathbf{x}$ we could compute the partial derivatives of L and use a gradient-based optimization method. However, this approach is not pursued in this article.

Instead, two methods of direct black-box optimization are used: the well known Nelder-Mead downhill simplex search and the evolutionary strategy with covariance matrix adaptation (CMA-ES).

Nelder-Mead simplex search. It is a well-known and established deterministic optimization algorithm [4]. During the search in D-dimensional space it maintains a set of $D+1$ points, forming the so-called simplex. The search is performed along a line which goes through the worst point of the simplex and the average of the other points. After a better point is found, it replaces a point in the simplex and the algorithm iterates. Because of the simplex behavior during the optimization, the algorithm is sometimes also called the *amoeba* algorithm.

Evolutionary Strategy with Covariance Matrix Adaptation. It is very recent and progressive stochastic optimization algorithm [5]. It maintains a D-dimensional normal distribution from which it samples new data points. The distribution is then in turn adapted based on the loss function values for these new points. The algorithm performs a kind of iterative principal component analysis of the selected perturbation vectors.

RESULTS

First, the proposed approach will be evaluated on synthetic data to decide which of these two algorithms (Simplex and Evolutionary algorithm) is more suitable for solving this particular optimization task. Then the selected method will be applied to real-world data.

SYNTHETIC DATA

The above described method was tested on artificially generated VOR signals to assess its success and precision and to decide which of the optimization algorithms is more suitable for this task. The tests were carried out on signals consisting of 1 to 5 sine components, i.e. the search was carried out in 2-, 4-, 6-, 8-, and 10-dimensional parameter spaces.

Generating VOR signal. First, for each sine component of the signal, the values of frequency, amplitude and phase shift were randomly generated. The ranges for individual parameters can be found in Table 1. Using these randomly generated values, a continuous MOS signal (which is to be estimated) is created. This signal then undergoes a disruption process which cuts it to individual segments with 'pauses' between them. This way the gaps created by filtering out the saccades are simulated. The segments are then placed to the same level (see Fig. 3, top).

Parameter	**Value (Range)**
f_i	$\langle 0.05, 2\rangle$
a_i	$\langle 0.2, 2\rangle$
ϕ_i	$\langle 0, \pi/2\rangle$
Sampling Freq.	500 Hz
Signal Duration	20 s
Saccade Duration	0.05 s

Table 1: Settings for parameters of artificial VOR signal

Experimental Evaluation. For each number of components, 9 different VOR signals were generated. For each of them the parameters of the underlying MOS were estimated by minimizing the loss function using both the Nelder-Mead simplex search and the CMA-ES. In each run, the algorithms were allowed to perform 10,000 evaluations of the loss function and a particular run was considered to be successful if the algorithm found a parameter set with the loss function value lower than 10^{-8}. The output from the algorithm should look like the one depicted in Fig. 3, bottom.

SUCCESS RATES

First, let us review the success rates of both algorithms when estimating the parameters of the MOS signal with the number of components ranging from 1 to 5 (see Table 2).

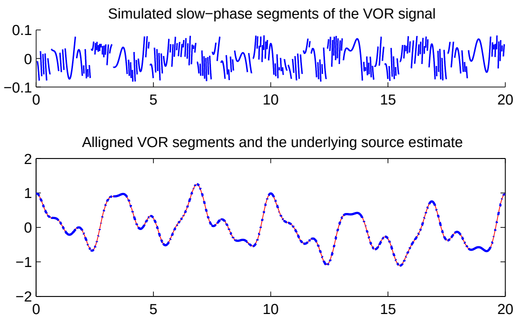

Figure 3: Top: Simulated VOR signal with saccades removed. This is the input of the algorithm. Bottom: VOR signal segments aligned with the estimated MOS signal. The parameters of the MOS signal are output of the algorithm.

Components	**Simplex**	**CMA-ES**
1	100.0	100.0
2	100.0	100.0
3	100.0	100.0
4	44.4	100.0
5	0.0	100.0

Table 2: Success rates (in percentages) of Simplex and CMA-ES algorithms

As we can see, the simplex algorithm has difficulties with finding the optimum of the loss function in less than 10,000 evaluations when the underlying MOS signal has 4 or more components.

SPEED OF THE ALGORITHMS

The comparison of speed is based on the number of evaluations needed to find a solution with loss value lower than 10^{-8}, i.e. only successful runs are considered. The results are summarized in Fig. 4. The two graphs reveal that the number of needed evaluations increases with the number of components (i.e. with the dimensionality of the search space) much faster for the simplex search method than for the CMA-ES where the increase is almost only linear (at least subquadratic). CMA-ES is clearly more scalable solution than the simplex search.

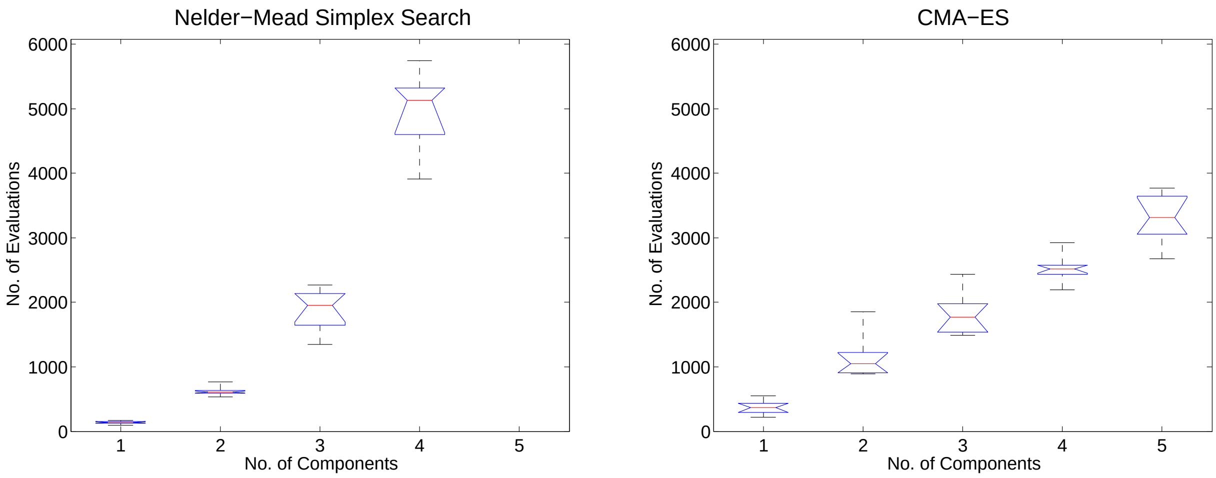

Figure 4: Number of evaluations needed to find a solution with quality better than 10^{-8} as a function of the number of components of the underlying MOS signal. *Middle line:* median, *box:* interquartile range, *whiskers:* minimum and maximum.

EVOLUTION PROFILES

The progress of evolution is depicted in Fig. 5. It presents the loss function value of the best solution found so far, averaged over all successful runs. Again, there is no line for the simplex method searching for parameters of 5 components.

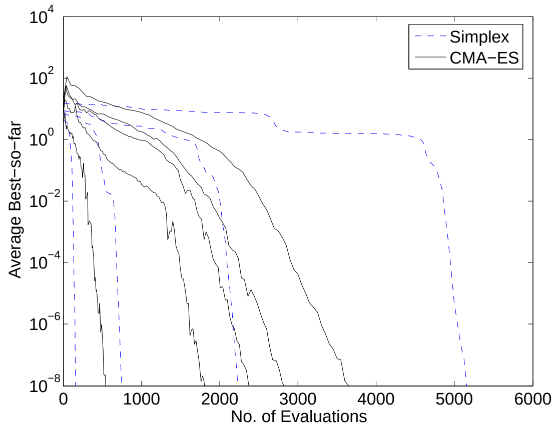

Figure 5: Typical progress of successful search runs as done by the simplex method and by the CMA-ES. Both the leftmost lines (solid and dashed) belong to 1 component, the rightmost dashed line belongs to simplex searching for parameters of 4 components while the rightmost solid line belongs to CMA-ES searching for parameters of 5 components.

Based on this graph, we could make a recommendation not to use the simplex search method when searching for parameters of the MOS signal with more than 2 components. The CMA-ES solves such tasks much better.

PRECISION OF THE ESTIMATES

The precision of the solutions provided by the algorithm must be assessed. We know that the segments of the simulated VOR signal come from a MOS signal with some specified parameters $\mathbf{x}' = (a'_1, \phi'_1, \ldots, a'_n, \phi'_n)$. The optimization algorithm provides the estimate of these parameters[1] $\mathbf{x} = (a_1, \phi_1, \ldots, a_n, \phi_n)$. The errors in estimates of the amplitudes are computed as

$$e_i^a = \frac{|a_i - a'_i|}{a'_i}, \tag{6}$$

and the errors in estimates of the phase shifts are computed as

$$e_i^\phi = \frac{|\phi_i - \phi'_i|}{\pi}. \tag{7}$$

The maximal error values for estimates of amplitudes and phase shifts across all components are presented in Tab. 3. The unsuccessful runs are excluded. That is also the reason of missing data for the simplex search with 5 components—there were no successful runs.

	Simplex		**CMA-ES**	
Comps	**max**(e^a)	**max**(e^ϕ)	**max**(e^a)	**max**(e^ϕ)
1	1.13e-4	5.71e-6	9.65e-5	2.21e-6
2	6.34e-5	8.41e-6	5.50e-5	5.49e-6
3	4.16e-5	5.74e-6	4.46e-5	6.57e-6
4	1.30e-5	4.11e-6	3.70e-5	1.17e-5
5	—	—	2.84e-5	3.43e-6

Table 3: Maximal errors for successful runs

Two direct search algorithms were compared. Although the simplex search is faster for signal with 1 or 2 sine components, it does not scale up well. For signals with 3 or more components, the CMA-ES is preferable—it is more

[1]It is assumed that these estimates are 'normalized', i.e. that all $a_i \geq 0$ and all $\phi_i \in \langle -\pi, \pi \rangle$.

robust, reliable and scalable. Based on these observations obtained for the simulated VOR signal, we decided to use the CMA-ES algorithm as the optimizer for the real-world signal.

REAL-WORLD DATA

When applying the proposed algorithm to real-world signals, the fast phases were first removed using thresholding approach. The resulting signal can be seen in Figure 6. In the next step, the CMA-ES method was applied to the signal. If we look at the result depicted in the left part of Fig. 7, it may seem that all the slow phases are aligned with the estimated MOS signal quite precisely. However, if we look at the picture in finer detail (see Fig. 7, right-hand side), we can see that the fit is not that good. The CMA-ES is misled by some fast phases that were not detected and were left in the signal after the thresholding procedure producing some kind of noise.

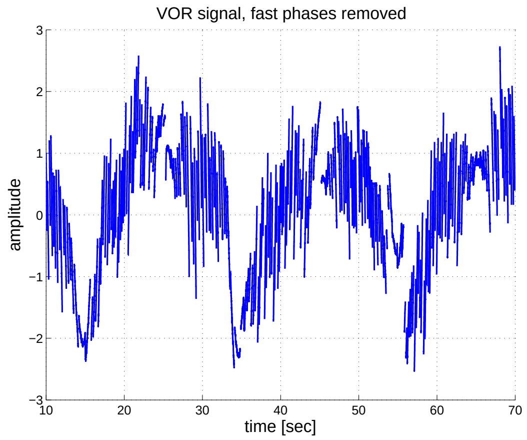

Figure 6: Real-World VOR signal with saccades removed.

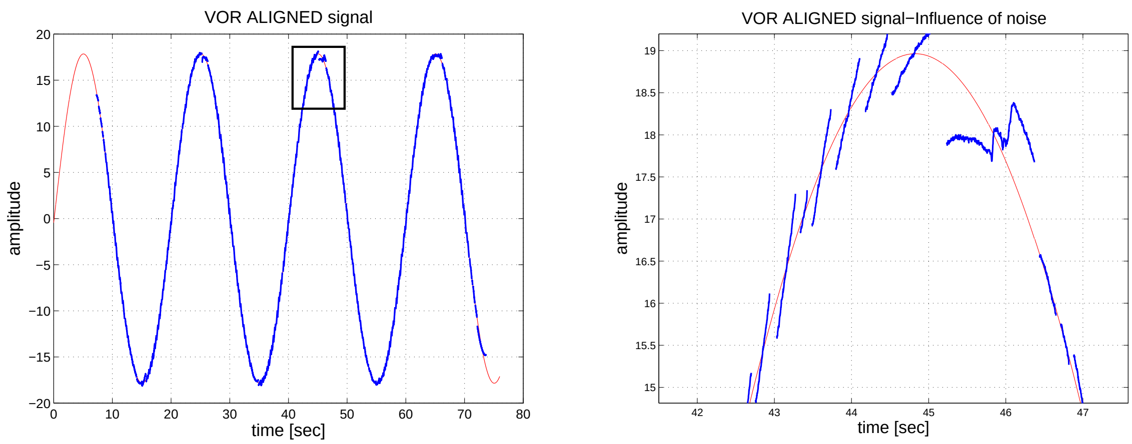

Figure 7: Real-World VOR signal with slow components aligned (left) and influence of presence of the non-identified saccades in the VOR signal (right).

CONCLUSIONS

In this paper, a new method of VOR signal processing was introduced and experimentally evaluated both on artificially generated signals and on clinical real-world data. It consists of two parts: (1) identification and removal of the fast phases, and (2) estimating the parameters of the underlying signal.

The method developed for saccades removal is simple and fast, and works well for the artificially generated signal, however, for real-world signals, it turned out that not all fast phases were detected. Depending on the sensitivity of the parameter estimation procedure, these ‘forgotten’ fast phases may significantly bias the estimates. The simplicity of the basic algorithms and its eficiency, even in the evaluation of irregular data such as patients suffering from nystagmus organ disorders, make it an interesting alternative for use in the clinical situations.

Regarding the estimation of the phase and gain parameters, conventionally used methods interpolate the signal segments and carry out the Fourier transform to obtain the amplification and the phase shift on the original frequencies. On the contrary, the proposed method directly estimates these parameters from the signal segments trying to align them with the underlying estimated mixture of sine waves. This way no artificial artefacts are introduced.

The future research must be aimed at improving the method for saccades removal because that seems to be the necessary condition for any successful algorithm for the vestibulo-ocular signal analysis.

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