Optimization of the Design of 3D Building Steel Structures Using Genetic Algorithms

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ABSTRACT: This paper shows the implementation of an elitist genetic algorithm (GA) that, when applied to 2D- and 3D- steel structures, is able to obtain structural elements with minimum weight and satisfy the safety factors or coefficients (Ultimate Limit States) of the applicable building code. To this end, an objective function has been defined that considers the constraints established by these coefficients. In addition, the codification of the design variables has been modified so that all of them have the same probability of initial selection; a selection operator has been implemented to consider the dispersion of the individuals within the population as well as a crossover operator that interchanges the sections assigned to the structural elements without their prior modification.

The final result is a robust genetic algorithm that is simple from a mathematical point of view and is able to work with complex structures under different load and constraint conditions. It does not need prior information about the objective function or the constraint functions and can work with complex structures under different load and constraint conditions. In addition, it permits the use of commercial sections catalogues as design variables and is able to apply the engineer experience in creacting groups of identical section, selecting these variables and their relation with the structural members.

The genetic algorithm developed by the authors is compared with common commercial solutions for complex structural optimization. Finally, the cost and weight improvements obtained and its trade-off with the higher computational cost, using the GA are discussed using some real cases. The result of the optimization is a 10% improvement on that obtained with conventional software.

KEYWORDS: Design optimization, Elitist genetic algorithm, Three-dimensional steel structures, Minimum weight, Ultimate limit states.

INTRODUCCIÓN

In general, the commercial structural analysis programs let optimize steel structures, although for that it is necessary the active intervention of the user. To avoid that, in 2002 the group of investigation of engineering projects developed an optimization module integrated in the Escal3D software [1] to perform the structural optimization automatically. Understanding as structural optimization a search of sections that with minimum weight can satisfy the ultimate limit states of the applicable building code given a fixed structural topology.
A previous step of the implementation of this module, was the analysis of the different structural optimization methods [2], [3]. From this one, the genetic algorithms were chosen as the most appropriate optimization methods, by the main features of genetic algorithms:

- They are not very complex from a mathematical point of view.
- They do not need an information about the objective function or the constraint functions so strictly as other optimization methods since they get this information in the form of fitness and penalties.
- They can work with any type of structure but with a fixed structural topology.
- They permit the use of discrete variables; in this case, the sections from commercial catalogues that are assigned to structural beams [4].

The following step was to develop an elitist genetic algorithm [5] in which different operators and processes were modified. The aim was to develop a genetic algorithm capable of obtaining structures of minimum weight able to support the demands that they might find themselves subject to. The main modifications carried out may be summarized as follows:

- A modified objective function that, applied to steel structures, minimizes the weight of the structure in accordance with the constrains due to the safety coefficients (Ultimate Limit States) [6].
- A new codification of the design variables so that they all have the same initial probability of being selected.
- A new crossover operator: *phenotype crossover*, capable of exchanging real sections obtained from commercial catalogues.
- A new selection operator: *aptitude*, which removes the worst individuals from the population.

To test the efficiency of the genetic algorithm, it was applied in two-dimensional structures and the results of this optimization were compared with the final structures obtained by means of a commercial structural analysis program. The result of this comparison was a 9.3% reduction in final weight of an articulated structure [7]. It may therefore be stated that, although the application of the elitist genetic algorithm supposes an increase in computational costs, the reduction in weight of the structures in two dimensions is high.
This paper presents the application of the elitist genetic algorithm to 3D building steel structures [8], although previously summaries the previous works carried out: modifications in the GA and the tuning of its parameters using a 2D structure.

PREVIOUS WORKS

CHANGES OVER THE GA

Formulation of the problem

The optimization problem may be expressed mathematically according to (1):

$$\begin{array}{lll} \textit{Minimize} & F(x) & \\ \textit{Constraints} & \dfrac{G_s(x)}{\tilde{G}_s(x)} \le 1 & s = 1,2,\ldots,s \\ & x = \left(x_1^T, x_2^T, \ldots, x_j^T\right) & j = 1,2,\ldots,j \\ & x_{i,j} \in D_j & \\ & D_j = \left(d_{j,1}, d_{j,2}, \ldots, d_{j,\lambda}\right) & \end{array} \qquad (1)$$

where the vector of the design variables x is divided into x_j sub-vectors, whose component $x_{i,j}$ take values from a D_j catalogue, and in which i is the number of design variables in each sub-vector and λ is the number of sections in each

catalogue, $G_s(x)$ is the calculated value of the constraint, $\tilde{G}_s(x)$ is its limited value and *s* is the number of inequality functions and finally *F(x)* is the objective function.
The interpretation of the equation is:
The aim of structural optimization, and in particular GAs, is to obtain the sections of the structural elements that minimize an objective, subject to certain limits or constraints.
These constraints may be classified in two types: explicit and implicit. Explicit constraints are analyzed without a simulation system. In contrast, implicit constraints like ours, require analysis and verification of the designs, such as for instance the allocation of areas to the sections. Although there are several methods of adjusting the constraints [9], the most appropriated method is therefore the penalizing of the members of a population that have one or more violations.
According to that and transforming the problem into nonrestrictive one, the mathematical expression is:

$$Minimize \quad \bar{F}(x,r) = F(x) + \bar{P}(r, G(x), H(x)) \tag{2}$$

where $\bar{F}(x,r)$ is the modified objective function and $\bar{P}(r,G(x),H(x))$ is the penalty term defined as a function of the penalty coefficient *r* and the constraint functions *G(x)* and *H(x)*.
The method is defined by means of penalty parameters, the rules that update these parameters and constraint functions.
As mentioned above, the aim of the implemented elitist GA is to obtain steel structures of minimum weight that fulfill the safety factors established by the applicable building code. This may be mathematically expressed according to (3).

$$\begin{array}{ll} Objective-function-to-minimize & F(x) = \rho \cdot \sum_{s=1}^{n_{bar}} x_s \cdot L_s \\ Constraints & G_s(x) \le 1 \quad H_s(x) \le 1 \quad \ldots \quad T_s(x) \le 1 \end{array} \tag{3}$$

where *F(x)* is the objective function (the weight of the structure analyzed), defined on the basis of ρ the density of the material, x_s the area of the section and L_s the length of n_{bar} bars that compose the structure; and the constraints, the limit values that the safety factors calculated in each bar $G_s(x)$, $H_s(x)$, ..., $T_s(x)$ may reach.
Applying the mathematical expression of the implemented GA (3) to the transformation of (2), the problem will be expressed mathematically according to (4).

$$\bar{F}(x,r) = F(x) + \sum_{s=1}^{n_{bar}} \left[r_1 \cdot G_s(x) + r_2 \cdot H_s(x) + \ldots + r_{n_c} \cdot T_s(x) \right] \tag{4}$$

where n_c is the number of safety factors established by the applicable building code.
Of the terms that define the modified objective function, the easiest to obtain is the weight, since it is directly defined on the basis of the geometric data of the structure, the characteristics of the material assigned to the bars and the properties of the sections assigned to these.
In contrast, in order to obtain the second term it is necessary to define the penalty coefficient and to carry out an analysis of the structure. In this way, the stresses and moments that define the safety factors and therefore the constraints of the problem will be obtained. The analysis of the structure and the verification of the safety factors can be carried out by means of a program of conventional analysis. In this study, the analysis was carried out using the Escal3D program [1], capable of obtaining the safety factors established by the Spanish building code [10] which defines safety factors such as the quotient between the calculated value and the maximum allowed value of: axial stress, shear stress, shear and bending stress, bending, Von Mises stress, buckling by compression, buckling by compression and bending, buckling by torsion, buckling by torsion and bending [6].
To define the penalty factor, it was considered the concept of safe bar like the one whose safety factors are equal to or lower than one. In addition, if the coefficient is far below unity the bar is considered *oversized*. That is to say, there exists another section with a smaller area that, if assigned to this bar, provides coefficients closer to unity, thus diminishing the weight of the structure.
In contrast, if the calculated safety factor is greater than unity, the bar is not able to support the stresses and moments calculated in it. In this case, it will be necessary to look for another section whose resistant properties are able to support these stresses and moments.
In accordance with the *safe structure* concept, the penalty coefficient is defined as the value which multiplied by the safety factor calculated in a bar, increases this coefficient if it is different from one and maintains it constant if it is equal to one. The sum of the penalized coefficients of all structural elements will be the penalty term of the modified objective function.
Therefore, the penalty term increases both, the weight of structures with bars that do not fulfill some of the safety factors and the weight of structures with oversized bars, distancing them, in both cases, from the sought-after minimum weight.

After several analysis, it was decided to carry out two types of adjustment of the penalty coefficient on the basis of whether the calculated safety factor was lower than unity or not. In the former case, an exponential distribution was followed, thus favoring the individuals with coefficients close to unity. In the later case, a linear distribution with penalty values much higher than the above values was followed in order to avoid the equality of weighting between oversized structures and "nonvalid" structures (5).

$$r(c)=\begin{cases} 0 & if \quad c=0 \\ e^{2-c}\cdot 10 & if \quad 0<c<1 \\ 1 & if \quad c=1 \\ c\cdot 1000 & if \quad c>1 \end{cases} \qquad c=G_s(x) \tag{5}$$

Encoding the design variables

The codification of design variables implemented [11] means that all the sections have the same probability of initial selection. This codification is represented in Figure 1, where the lengths of chain of n bits present n_{pos} possible sections as opposed to the n_{ex} existing sections in the commercial catalogue.
The proposed solution consists in checking whether the number assigned randomly to each design variable has an empty position assigned to it or, on the contrary, it has a real section assigned to it. In the former case, the design variable with an empty position is generated again until a real section is assigned to it, while in the latter case this section will now become part of the population. In this way, it not only assures that all the sections assigned to the structural elements belong to the catalogue of commercial sections but also that all of them have the same probability of initial selection.

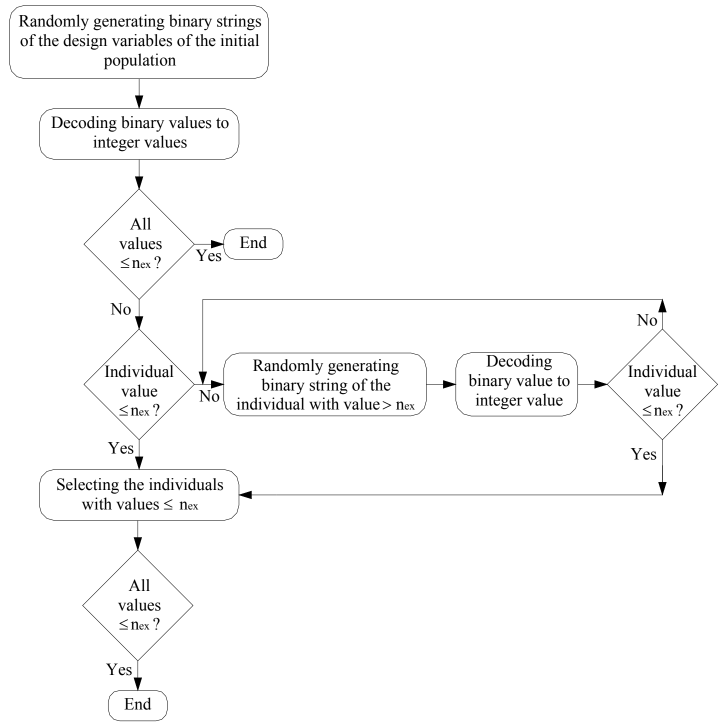

Figure 1: Encoding the design variables.

Phenotype crossover.

In general, GAs use crossover operators that interchange bits, randomly assigning the crossover points [11]. This entails not only an exchange of information, but also an alteration in the design variables.
This modification is avoided by means of the so-called phenotype crossover. In this operator, the crossover point is located between two phenotypes or design variables from two individuals termed parents. Two new strings, the

children, are created swaping all the characters between the selected position and the overall length of the parents strings (Figure 2).

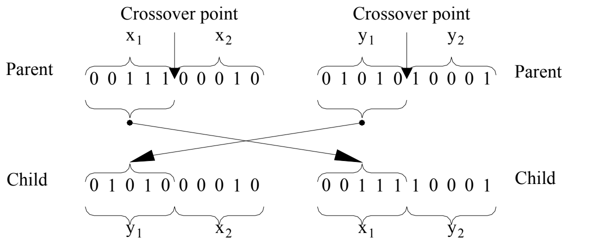

Figure 2: Phenotype crossover in binary representation.

Aptitude selection operator.

In the most frequent reproduction found in the bibliography [12], an individual is selected to form part of the new generation based on its fitness, independently of whether this is very far or not from the average. This can give rise to isolated individuals or “strangers”, though with high aptitudes, also having a high number of offsprings, thus vastly altering subsequent generations.

For this reason, a reproductive operator denominated *aptitude* [11] has been implemented that considers the population dispersion. In this operator:

- A new function is defined on the basis of the modified objective function, denominated aptitude function.
- The value of the aptitude function or aptitude of all the individuals is obtained and those whose values are lower than the average are eliminated.
- A probability of rejection is defined for the surviving individuals, of inverse value to aptitude.

The new population is thus created from the best individuals of the previous population, avoiding isolated elements and increasing the speed of the GA in the search for the optimal individual.

FLOW OF THE ELITIST GA DEVELOPED

The flow of the implemented elitist GA is represented in Figure 3.

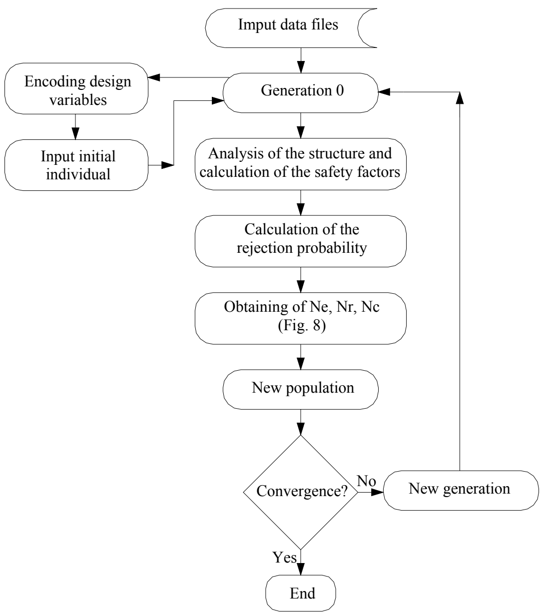

Figure 3: Elitist GA flow.

According to the Figure 3, each new population is formed by three types of individuals obtained from the surviving population:

1. Elite individuals, selected between the best individuals of the current population without mutation and whose number N_e is the result of multiplying the elite probability P_e and the population size N_p.
2. Crossover individuals, selected between the surviving individuals in function of their probability of rejection with mutation and whose number N_c is the result of multiplying the crossover probability P_c and the population size N_p.
3. Random individuals, whose number is equal to the difference between the total number of individuals in the population and the sum of elite and crossover individuals.

TUNING OF THE GA PARAMETERS

To tune the parameters of the GA [13] (N_p, P_e, P_c, P_{mut}), it was carried out a study, on the structure shown in the Figure 4, using five runs for each combination of the domain indicated in the Table I. The conclusions of this study can be summarized as follows:

- When the value of N_p increases, the average weight of the best individuals decreases, although the average number of evaluations increases too. It can be observed that solutions with a good performance of the developed GA are obtained with a value of N_p between 60 and 100 individuals.
- A value of P_{mut} ranging between 1% and 3% and a value of P_e ranging between 10% and 30% give better solution achieved within a reasonable average number of function evaluations and coefficients next to one.
- An increase of the P_c until values of 80%, with the same probability of elite, decreases the average weight.

N_p	P_e	P_c	P_{mut}
20 -140	0.0 – 90%	10 – 90%	0.1 – 4%
	$P_c + P_e \leq 100\%$		

Table I: Domain of parameters.

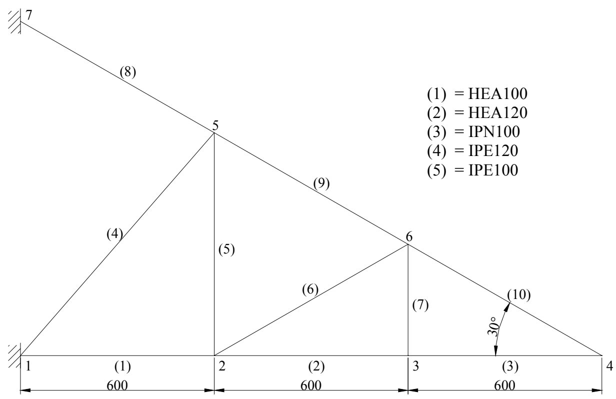

Figure 4: Structure analysed in the tuning of the parameters.

EFFICIENCY OVER 2D STEEL STRUCTURES

In order to test the suggested improvements (in addition to the modification of the penalty factor), and after tuning the GA parameters, it was carried out an study in order to find the best individual over the structure above mentioned. This one was found in 100 runs with the following parameters: $N_p = 100$, $P_e = 30\%$, $P_c = 70\%$, $P_{mut} = 1\%$.

The steel sections obtained in this individual and its minimum weight are given in Table II. This table also includes the solution obtained from a conventional design approach, using a commercial program of structural analysis. It can be seen that the minimum weight solution is about 9,3% lighter than the conventional design.

Number of group	Sections (E.G.A.)	Weight (kg)	Sections (conventional design)	Weight (kg)
1	HEA300	1519,14	HEA300	1519,14
2	HEA300	773,50	HEA300	773,50
3	LSI150x15	345,08	IPN240	378,60
4	HEB220	488,49	IPE550	715,15
5	IPN400	1935,72	IPE550	2145,45
Weight (kg)		5061,93		5531,84

Table II: Best individual against conventional design.

OPTIMIZATION OF 3D BUILDING STEEL STRUCTURE

ANALYZED STRUCTURE

To the study, one of the most common 3D structures was chosen: a three-floor *steel building*, more and more common in civil construction (Figure 5), with a separation of 4 m between floors and 6 m between columns.

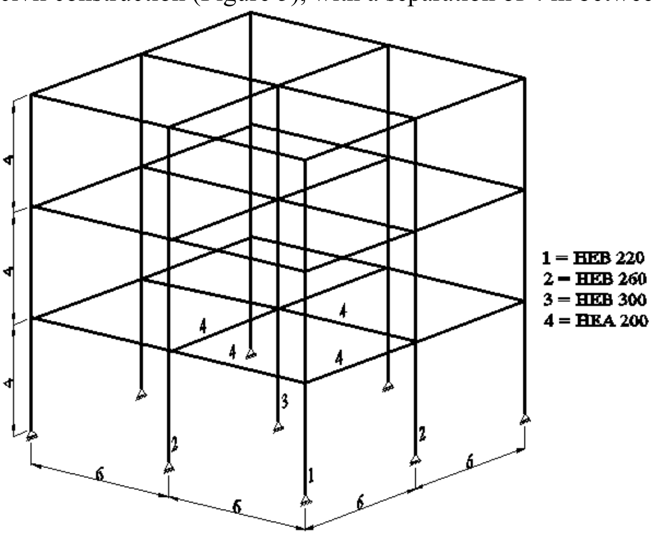

Figure 5: Steel building analysed.

The load hypotheses that are applied to the structure are calculated following the Spanish Basic Building Code (NBE EA-95) [10]: the weight of the structure itself, live load due to wind and use and a combination of both.

SECTIONS DATABASE

The sections database used in the initial tests carried out with 3D structures was that employed in the study of 2D structures [7]. This database of 2835 elements contains all the sections from the Spanish building code (NBE EA-95). Unlike its counterparts in other countries, the Spanish building code allows greater flexibility in the assignment of sections to beams, since it does not specify which sections must be assigned to vertical or horizontal elements.
But, the first results of the optimization processes in 3D structures show that, the sections that are assigned to the final individuals of each optimization fall within a smaller range of sections: the series of sections IPN, IPE, HEA, HEB, HEM [6]. In Spanish steel buildings, IPN and IPE sections are generally used in horizontal elements while the rest are used in vertical elements. Just like in the English code, where UB and UC sections are used for horizontal and vertical elements respectively [14].

For this reason, it was decided to reduce the database to 114 sections. The computational cost is also reduced due to the use of less chains to use in all the optimization process, besides it is easier to find a local minimum, and finally the empty positions in the database are reduced.

ASSIGNMENT OF INITIAL SECTIONS

Once the database to be used had been analyzed, the first sections (Figure 5) were assigned to the beams of the structure, grouping these according to:

1. The loads applied to the structure.
2. Their location.

These two concepts mark the type of demand that prevails on beams; i.e. whether they are subject to bending, torsion, shear, etc. Given that the behavior of sections from the viewpoint of stress depends on their resistance properties (second moment of area, etc.), the experience of the designer is fundamental when carrying out an initial assignment of the sections of the structure.

ANALYSIS OF THE RESULTS

Five complete evolutions were carried out in this study for the analyzed structure (Figure 5) starting out from the same initial individual, thus giving rise to five different optimum individuals. These differences are due to the fact that, as occurs in natural evolution, the generations in genetic algorithms are the fruit not only of the crossover between individuals but also of mutation.
The study of the sections assigned to the five optimum individuals of the structure (Table III) was performed in two steps: a first analysis in which each individual was analyzed independently from the rest, and a second step in which the individuals were compared with one another.
According to the Table III, there exist hardly any similarities among the sections assigned to the different groups with the same individual. But from the analysis of the sections assigned to the same groups in different individuals, it can be observed that the sections assigned to the horizontal structural elements, which constitute group IV, are repeated throughout four out of the five evolutions. However, the sections assigned to the vertical structural elements, groups I, II and III, are barely repeated in two or three individuals. This is due fundamentally to the fact that the load of the structure is shared out among the vertical structural elements, which means that its sections depend strongly on one another. That is to say, when the section and hence the strength capacity of one of the groups II, III or IV is reduced, the section of the remaining two must be increased in order to compensate the loss in support capacity.

	Group I	Group II	Group III	Group IV
Individual 1	IPN320	HEA260	HEA240	IPE360
Individual 2	IPN300	IPE360	HEM140	IPE360
Individual 3	HEB220	HEA260	HEA240	IPE360
Individual 4	HEA200	HEA260	HEA260	IPE360
Individual 5	HEA300	HEA240	HEM140	IPE330

Table III: Sections of the optimum individuals.

The optimum individual given in each optimization process could be a local optimal solution. Since it is impossible to know that beforehand, it is necessary to run several processes and choose the best among all of the intermediate optimum structures.
In the selection of the final optimum individual, however, not only do the sections assigned to the groups have an influence, but also:

- Their <u>final weight</u>, understood as the weight of all the sections that make up the structure.
- The <u>penalty</u> that resulting safety coefficients produce, because they are taken into account by the modified objective function as another weight [5].
- The <u>number of evaluations</u> that must be carried out to reach said optimum individual; a value that indicates how many structures are generated throughout optimization and therefore how many are analyzed, with the corresponding computational cost.

From the analysis of the Figure 6 and Figure 7, a relationship may be appreciated between the weight of the beams and the penalty factor. In general, a lower weight of beam coincides with a lower weight of the penalty factor. In contrast, however, the weight of the beams and the number of evaluations performed are inversely related, since a lower weight of beam coincides with a greater number of evaluations.

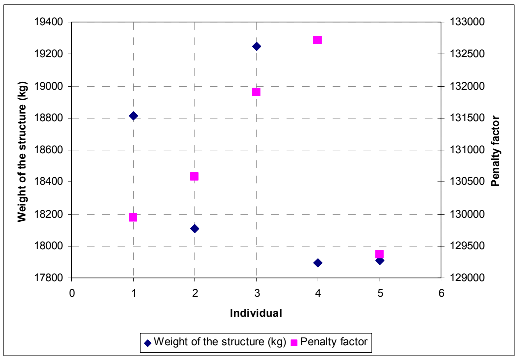

Figure 6: Weight of the steel building against penalty factor in the five evolutions of the optimization process.

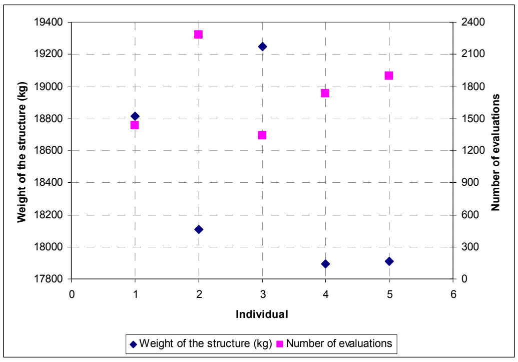

Figure 7: Weight of the steel building against number of evaluations in the five evolutions of the optimization process.

COMPARISON OF THE RESULTS

In order to test how good the optimization process is, one of the optimum individuals obtained from this process was chosen and compared on the one hand with the starting-out structure to which the designer had assigned some sections, and on the other to the individual obtained from the optimization carried out using a conventional matrix analysis program.

The fifth individual has been chosen as the best individual, owing to the fact that, at similar weights of individuals 4 and 5, the penalty factor of the latter is lower (Figure 6).

When the initial individual is compared with the optimum individual, an increase of 0.25% is observed. In order to understand this behavior, it is necessary to jointly analyze the initial structure, the structure obtained in the optimization process and that obtained using the conventional design program. As can be seen (Table IV), the sections assigned by the conventional software are similar to the initial structure, except for group V, in which the section has been increased. This indicates that the profile of this group was not initially capable of supporting the stresses to which it was subjected. Due to the fact that one section does not comply with some safety coefficient, the structural optimization produces a change in all the sections in this structure, thus obtaining a combination of these that increases the weight a little but which is able to comply with all the safety coefficients. What is more, the result of the optimization is a 10% improvement on that obtained with conventional software.

Number of group	Sections (initial individual)	Weight (kg)	Sections (E.G.A.)	Weight (kg)	Sections (conventional design)	Weight (kg)
I	HEB220	3384.38	HEA300	4051.01	HEB220	3384.38
II	HEB260	4344.43	HEA240	2787.87	HEB260	4344.43
III	HEB300	1365.61	HEM140	771.65	HEB300	1365.61
IV	HEA200	8771.33	IPE330	10299.58	HEA220	10574.44
Weight (kg)		17865.75		17910.11		19668.86

Table IV: Initial individual against the best individual and against the individual from the conventional design.

CONCLUSIONS

From the results obtained when applying the Elitist Genetic Algorithm to 3D structures, it may be concluded that:

- The final structure presents sections with safety coefficients very close to unity, i.e. the assigned sections have the sufficient strength to support the stresses but with a minimum weight.
- The different combinations of sections obtained for one single structure allow us to choose the most adequate one from a building perspective or the one whose sections are most easily obtained.
- The optimization process may be used to group together the elements of a structure in groups with the same section.
- The weight of the structure is lower than that obtained using conventional software, due fundamentally to the random assignment of sections.
- The elitist genetic algorithm is appropriated for the optimization of 3D structures.

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